Comparison between different control strategies for estimation purposes using Control-based Observer paradigm

Andrei Popescu, Gildas Besançon, Alina Voda
Univ. Grenoble Alpes, CNRS, Grenoble INP, GIPSA-lab, 38000 Grenoble, France.
Emails: (andrei.popescu, gildas.besancon, alina.voda)@gipsa-lab.grenoble-inp.fr

Institute of Engineering Univ. Grenoble Alpes

***Abstract*—This paper presents a comparison between different control strategies used for estimation purposes for systems with unknown inputs in the recently proposed Control-based Observer approach. In particular, several robust state and unknown inputs observers are derived using classical controllers such as Proportional or Proportional Integral, Linear Quadratic Regulator or Linear Quadratic Integrator and H_∞ Full information. They are also compared with criteria such as the quality of the estimation, the speed of the observers, the robustness against the noise and the complexity of the observers. Simulation results are provided to better highlight the comparison.**

Keywords - *Observers for linear systems, unknown inputs estimation, control-based observer, P, PI, Linear Quadratic Regulator, Linear Quadratic Integrator and H_∞ Full Information control strategies*

I. Introduction

The observer problem is of great importance in the field of automatic control, motivated mainly by the need to estimate some *internal information* describing a dynamical system, which is not available, by using some *external measurements*. There are multiple reasons for which one cannot directly access some internal information such as technological constraints (some quantities cannot be measured) or economical ones (the cost of the sensors can be quite high). The importance of an observer can be easily explained by its central role in automatic control applications. Various purposes can be associated to it such as monitoring (fault detection), modeling (parameters identification) or control (state estimation). A long and rich history stands behind solving the observer problem. Among the solutions proposed we can find classical methods like Luenberger observer [1] or Kalman filter [2]. An extension towards robust solution for the above mentioned problem is the H_∞ observer [3]. A more simple alternative approach for a robust state estimation is obtained using PI observer [4].

In the present paper, a different approach to design observers is taken into account following a technique recently introduced in [5] which proposes that instead of solving the direct observer problem, one can design an observer by solving a control problem. This new paradigm is called *Control-based Observer* (CbO) design and the idea behind it states that one can design a control law for a model of the system such that the real output of the system is followed by the output of the model. The method already showed good results for different applications such as wind speed estimation [6], surface estimation in Scanning Tunneling Microscope [7], [8] [9] and unknown input disturbance estimation in a magnetic levitation process [10].

A general problem that can be addressed in this framework can be formulated as follows: given a linear system with known and unknown inputs, design an observer for state and unknown inputs estimation based on the known inputs and outputs of the system, the model of the system, and a chosen control strategy such that the outputs of the model follow the system outputs.

The main contribution of the paper is to present a new robust Control-based Observer based on an H_∞ Full Information control design, together with a comparison with different other control approaches such as a Proportional (P), Proportional Integral (PI), Linear Quadratic Regulator (LQR) and Linear Quadratic Integrator (LQI) designed for the same goal. The different control strategies used for estimation purposes in CbO framework are compared regarding criteria such as the quality of the estimation, the speed of the observers, the robustness against the noise and the complexity of the obtained observers to illustrate the performances of the observers.

The paper is organized as follows: in Section II the Control-based Observer paradigm is recalled highlighting different formulations of the problem which can be solved. In Section III the analytic solutions of the observers which will be compared are given, including the H_∞ based design. Simulation results are presented in Section IV and finally Section V concludes the paper.

II. Control-based Observer paradigm

This section briefly describes the principle of Control-based Observer, in particular for linear systems having unknown inputs. The idea for this technique has its roots in the duality between observer problem and control problem.

Let us consider a linear system described by:

$$\begin{aligned} \dot{x} &= Ax + Bu + Gv \\ y &= Cx + Du + Jv \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ and $v \in \mathbb{R}^q$ classically stand for state, known input, measured output and unknown inputs of the system. Moreover, matrices A, B, C, D, G, J completely describe the state and the output evolution of the linear system.

The Control-based Observer approach states that one can design an observer by controlling a model of the system computing an appropriate control input such that the output of the model follows the output of the system (tracking problem) as described in the following equation:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + G\hat{v} \\ \hat{y} &= C\hat{x} + Du + J\hat{v} \\ \hat{v} &= \kappa(\hat{x}, t) \end{aligned} \tag{2}$$

where first equation in (2) represents the dynamic of the model, $\hat{y}$ is the output of chosen model and finally $\hat{v}$ is the control law described by the controller $\kappa(\hat{x}, t)$.

One can notice that the model of the system is controlled via the unknown inputs such that $\hat{y}$ follows y, thus as a consequence the observer can provide not only the state of the system, $\hat{x}$ which will be an estimate of x, but also the unknown inputs, $\hat{v}$ which will be an estimate of v.

It is worth mentioning at this point that in order to estimate the state and the unknown inputs the system described by equation (1) has to fulfill some appropriate observability conditions while the model described by (2) has to admit some appropriate controllability requirements.

This principle is summarized in Figure 1.

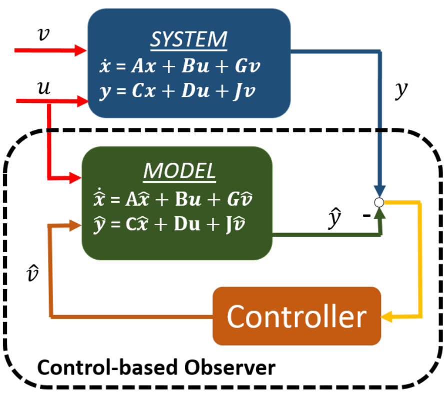


Fig. 1: Control-based Observer principle

Following this brief description of Control-based Observer approach it is clear how the observer problem can be converted into a control (tracking) problem.

However depending on the estimation problem which has to be solved the observer obtained using the control-based approach can have different structures. For example, one can formulate the objectives of the estimation problem concerning:

i. State estimation
ii. Input estimation
iii. State and input estimation

Another formulation could be seen regarding the information available for the controller:

i. Output feedback - the information available is the error between $\hat{y}$ and y
ii. State feedback - the information available is the full state of the model, $\hat{x}$ (and of course the output y)

III. Designing different controllers for CbO

In this section we exploit the choice of different controllers for the control-based framework. In particular we start with some simple control strategies such as P and PI controller, followed by some more advanced techniques such as LQR and LQI controllers and finally a robust H_∞ Full Information controller is presented.

As for the estimation problem that we want to solve we consider a system which includes unknown inputs. The objective is to estimate the state as well as the unknown inputs of the system. The controllers designed for Control-based Observers use the information about the state of the model and about the output of the system.

In this section we consider the system described in equation (1) further including state noise, $w \in \mathbb{R}^r$, and measurement noise, $n \in \mathbb{R}^s$, as follows:

$$\begin{aligned} \dot{x} &= Ax + Bu + Fw + Gv \\ y &= Cx + Du + Hn + Jv \end{aligned} \tag{3}$$

In order to design a CbO for estimating the state and the unknown inputs it is assumed that the following conditions regarding the system described by equation (3) hold:

a1. The pair $\left(\begin{bmatrix} A & G \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} C & J \end{bmatrix}\right)$ is observable

a2. The pair (A, G) is controllable

where the first conditions ensures the vector $\left[x^T v^T\right]^T$ can be reconstructed, while the second one ensures that the model can be controlled such that the output of the model follows the output of the system.

Next, the control strategies chosen to design the Control-based Observer are presented. Without loss of generality, we will consider further on that the matrices $D = 0$ and $J = 0$.

A. P and PI controllers

1) P controller: For this particular problem one can choose a two-degree control strategy: simple state feedback and the output y of the system which has to be followed by $\hat{y}$:

Model:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + G\hat{v} \\ \hat{y} &= C\hat{x} \end{aligned} \tag{4}$$

Control strategy:

$$\hat{v} = -K_x^P \hat{x} + K_y^P y \tag{5}$$

where K_x^P is chosen such that $A - GK_x^P$ in equation (6) is stable and has a certain dynamic (a pole placement technique can be applied) and $K_y^P = [C(A - GK_x^P)^{-1}G]^{-1}$ to ensure low frequency accuracy.

Observer:

$$\dot{\hat{x}} = (A - GK_x^P)\hat{x} + Bu + GK_y^P y \tag{6}$$

Clearly, the dynamic of the observer in this case is given by the matrix $A - GK_x^P$ which can be chosen arbitrarily fast and as soon as $\hat{y}$ follows y we can obtain $\hat{x}$ the estimate of state system x and $\hat{v}$ the estimate of unknown input v.

Remark:

Notice that one can recover the Luenberger Observer using this Control-based Observer framework by considering the case for the system model with no unknown input and choosing as control strategy a simple 'output feedback' P controller (K^P) as described in the following equation:

$$\dot{\hat{x}} = A\hat{x} + Bu + GK^P(\hat{y} - y) \tag{7}$$

Since the only constrain for choosing G is that the pair (A, G) is controllable, G can be set to be the identity matrix, which finally leads to the equation of Luenberger Observer:

$$\dot{\hat{x}} = (A - K^P C)\hat{x} + Bu + K^P y \tag{8}$$

2) PI controller: A PI controller is also chosen to control the model in order to assure that $\hat{y}$ follows y. An extension of the system model is needed to include the integral effect of the controller which leads to the following equations.

Model:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + G\hat{v} \\ \dot{\hat{x}}_i &= \hat{y} - y \\ \hat{y} &= C\hat{x} \end{aligned} \tag{9}$$

Control strategy:

$$\hat{v} = -K_x^{PI}\hat{x} - K_i^{PI}\hat{x}_i \tag{10}$$

In order to compute K_x^{PI} and K_i^{PI} let us consider:

$$K^{PI} = \begin{bmatrix} K_x^{PI} & K_i^{PI} \end{bmatrix} \tag{11}$$

and set:

$$A^{PI} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \quad G^{PI} = \begin{bmatrix} G \\ 0 \end{bmatrix} \tag{12}$$

Finally, K^{PI} is computed such that $A^{PI} - G^{PI}K^{PI}$ is stable and has an imposed dynamic (a pole placement technique can be again applied).

Observer:

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{x}}_i \end{bmatrix} = \begin{bmatrix} A - GK_x^{PI} & -GK_i^{PI} \\ C & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{x}_i \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} y \tag{13}$$

The dynamic of the observer is given by the matrix $A^{PI} - G^{PI}K^{PI}$ which can again be chosen arbitrarily fast and as soon as $\hat{y}$ follows y we can obtain $\hat{x}$ the estimate of state system x and $\hat{v}$ the estimate of unknown input v.

Remark:

Notice that another classical observer can be obtained in this case, the so called PI observer. Again the case for the system model with no unknown input is considered together with an 'output feedback' PI controller as presented in the equation below:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + GK^{PI}(\hat{y} - y) + GK_i^{PI}x_i \\ \dot{\hat{x}}_i &= \hat{y} - y \end{aligned} \tag{14}$$

Setting again G to the identity matrix leads to the equations of PI Observer.

B. LQR and LQI controllers

1) LQR controller: The discussion continues with the choice of a Linear Quadratic Regulator (LQR) as a control strategy.

Model:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + G\hat{v} \\ \hat{y} &= C\hat{x} \end{aligned} \tag{15}$$

Control strategy:

$$\hat{v} = -K_x^{LQR}\hat{x} + K_y^{LQR}y \tag{16}$$

where $\hat{v}_1 = -K_x^{LQR}\hat{x}$ is computed by minimizing a cost function of the form:

$$J_{LQR} = \int_0^\infty (\hat{x}^T Q_{LQR}\hat{x} + \hat{v}_1^T R_{LQR}\hat{v}_1)dt \tag{17}$$

where Q_{LQR} and R_{LQR} are positive semidefinite and definite matrices.

Finally, the feedback matrix K_x^{LQR} is given by the equation:

$$K_x^{LQR} = R_{LQR}^{-1}G^T X_{LQR} \tag{18}$$

Where X_{LQR} is the solution of Algebraic Riccati Equation:

$$A^T X_{LQR} + X_{LQR}A + K_x^{LQR^T} R_{LQR} K_x^{LQR} + Q_{LQR} = 0 \tag{19}$$

Here again, K_y^{LQR} can be computed so as to reduce low frequency error between y and $\hat{y}$.

Observer:

$$\dot{\hat{x}} = (A - GK_x^{LQR})\hat{x} + Bu + GK_y^{LQR}y \tag{20}$$

2) LQI controller: The next controller chosen is a Linear Quadratic Integrator and again the extension of the model is considered by adding the integral action. Model:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + G\hat{v} \\ \dot{\hat{x}}_i &= \hat{y} - y \\ \hat{y} &= C\hat{x} \end{aligned} \tag{21}$$

Control strategy:

$$\hat{v} = -K_x^{LQI}\hat{x} - K_i^{LQI}\hat{x}_i \tag{22}$$

To compute the gain of the Linear Quadratic Integrator controller, $K^{LQI} = \begin{bmatrix} K_x^{LQI} & K_i^{LQI} \end{bmatrix}$, let us consider the following state space representation for the extended system ($x_{LQI} = \begin{bmatrix} \hat{x} \\ x_i \end{bmatrix}$):

$$A^{LQI} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \quad G^{LQI} = \begin{bmatrix} G \\ 0 \end{bmatrix} \tag{23}$$

where we want to compute $\hat{v} = -K^{LQI} x_{LQI}$ by minimizing a cost function:

$$J_{LQI} = \int_0^\infty (x_{LQI}^T Q_{LQI} x_{LQI} + \hat{v}^T R_{LQI} \hat{v}) dt \tag{24}$$

where Q_{LQI} and R_{LQI} are positive semidefinite and definite matrices.

Finally, the feedback matrix K^{LQI} is given by the equation:

$$K^{LQI} = R_{LQI}^{-1} G^T X_{LQI} \tag{25}$$

Where X_{LQI} is the solution of Algebraic Riccati Equation:

$$A^{LQI^T} X_{LQI} + X_{LQI} A^{LQI} + K^{LQI^T} R_{LQI} K^{LQI} + Q_{LQI} = 0 \tag{26}$$

Observer:

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{x}}_i \end{bmatrix} &= \begin{bmatrix} A - GK_x^{LQI} & -GK_i^{LQI} \\ C & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{x}_i \end{bmatrix} \\ &+ \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} y \end{aligned} \tag{27}$$

C. H_∞ Full Information controller

Finally let us present the H_∞ Full Information Controller approach (see for example [11]) to obtain a robust Control-based Observer. Notice that this provides a less complex structure for the observer (lower dimension) than in our former approach of [8].

In particular indeed, for the Full Information problem the corresponding generalized plant has a special form, because it is assumed that the state of the generalized plant, $\hat{x}_{FI}$, as well as all external signals, y, are known, which means that the controller is provided with Full Information.

Let us first consider the block diagram of the observer as illustrated in Figure 2: The generalized plant of the control

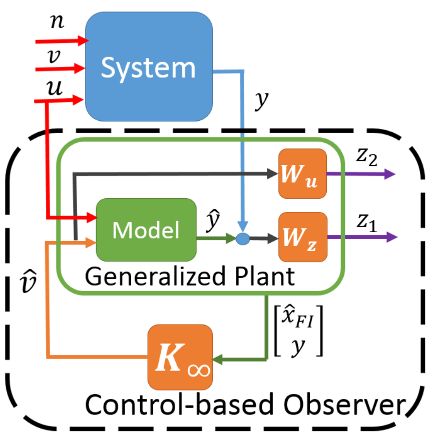


Fig. 2: Control-based Observer - H_∞ FI controller

problem is:

Model:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + G\hat{v} \\ \dot{x}_{ps} &= A_{1ps}\hat{x} + A_{2ps}x_{ps} + B_{1ps}y + B_{2ps}\hat{v} \\ z &= C_1 \begin{bmatrix} \hat{x} \\ x_{ps} \end{bmatrix} + D_{11}y + D_{12}\hat{v} \end{aligned} \tag{28}$$

where $x_{FI} = \begin{bmatrix} \hat{x} \\ x_{ps} \end{bmatrix}$ is the state of the generalized plant, y is the external signal, $\hat{v}$ is the control input and z is the error signal which has to be kept small. The second equation of (28) represents the performance specification of the control problem, $x_{ps} \in \mathbb{R}^k$. The state dimension of this equation depends on the complexity of the chosen templates.

Control strategy:

$$\hat{v} = -K_x^\infty \begin{bmatrix} \hat{x} \\ x_{ps} \end{bmatrix} - K_y^\infty y \tag{29}$$

where the goal is to search for $K_\infty = \begin{bmatrix} K_x^\infty & K_y^\infty \end{bmatrix}$ such that given an attenuation $\gamma > 0$ we have:

$$||T_{zy}(s)||_\infty = \frac{||z||_2}{||y||_2} < \gamma \tag{30}$$

Let us consider the following notations:

$$A_{FI} = \begin{bmatrix} A & 0 \\ A_{1ps} & A_{2ps} \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ B_{1ps} \end{bmatrix} \quad B_2 = \begin{bmatrix} G \\ B_{2ps} \end{bmatrix} \tag{31}$$

The assumptions relevant to the Full Information problem concerning the generalized plant are :

A1. (A_{FI}, B_2) is stabilizable

A2. D_{12} is full column rank

A3. $\begin{bmatrix} A_{FI} - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all ω

The solution for H_∞ Full Information problem is given in terms of some Algebraic Riccati Equation solution for which the corresponding Hamiltonian matrix is:

$$\begin{aligned} H_{FI} &= \begin{bmatrix} A_{FI} & 0 \\ -C_1^T C_1 & -A_{FI}^T \end{bmatrix} \\ &- \begin{bmatrix} B \\ -C_1^T D_{1\bullet} \end{bmatrix} R_{FI}^{-1} \begin{bmatrix} D_{1\bullet}^T C_1 & B \end{bmatrix} \end{aligned} \tag{32}$$

where

$$\begin{aligned} R_{FI} &= D_{1\bullet}^T D_{1\bullet} - \begin{bmatrix} \gamma^2 I & 0 \\ 0 & 0 \end{bmatrix} \\ D_{1\bullet} &= \begin{bmatrix} D_{11} & D_{12} \end{bmatrix} \qquad B = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \end{aligned}$$

Assuming that the appropriate Algebraic Riccati Equation described by the Hamiltonian, H_{FI}, defined above has a stabilizing solution (denoted by X_∞) we define:

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = -R_{FI}^{-1} \left[D_{1\bullet}^T C_1 + B^T X_\infty \right] \tag{33}$$

$$K_\infty = K_x^\infty x_{FI} - K_y^\infty y \tag{34}$$

for which we have the gains:

$$K_x^\infty = \begin{bmatrix} T_2 & I \end{bmatrix} F \quad K_y^\infty = T_2 \quad T_2 = D_{12}^T D_{11} \tag{35}$$

Observer:

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}} \\ \dot{x}_{sp} \end{bmatrix} &= \left(\begin{bmatrix} A & 0 \\ A_{1ps} & A_{2ps} \end{bmatrix} - \begin{bmatrix} G \\ B_{2ps} \end{bmatrix} K_x^\infty \right) \begin{bmatrix} \hat{x} \\ x_{ps} \end{bmatrix} \\ &= \begin{bmatrix} B \\ 0 \end{bmatrix} u + \left(\begin{bmatrix} 0 \\ B_{1ps} \end{bmatrix} - \begin{bmatrix} G \\ B_{2ps} \end{bmatrix} K_y^\infty \right) y \end{aligned} \tag{36}$$

IV. Simulation results and comparison among the different Control-based Observers

A. Simulation system parameters

In this section, a linear second order system having unknown inputs is proposed in order to test and compare all the Control-based Observers presented. It is in particular representative of the former real applications that we addressed with this Control-based Observer [6]-[9] and can easily highlight the key components of the estimation method as well as the performances and the complexity of the designed observers.

Let us consider the system as described in the equation:

$$\begin{array}{rcl} \dot{x} & = & Ax + Fw + Gv \\ y & = & Cx + Hn \end{array} \tag{37}$$

where the variables have the same meaning as in equation (3). In particular, we have:

$$A = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ g_1 \end{bmatrix} \quad C = \begin{bmatrix} c_1 & 0 \end{bmatrix} \tag{38}$$

Where all the coefficients of the matrices which describe the system are unitary. One can notice that the input, u, is here omitted. This doesn't change the observer structure since the system is linear and u is assumed to be known.

Next the Control-based Observers using LQR, LQI and H_∞ FI controllers are designed. We don't consider for comparison, the simple cases of a P and PI controller since in the presence of measurement noise, the performances of those observers become quite poor.

On the one hand, to design the CbO using LQR regulator we have chosen the weighting matrices $Q_{LQR} = \begin{bmatrix} 10^2 & 0 \\ 0 & 10^2 \end{bmatrix}$ and $R_{LQR} = 1$. On the other hand, for the LQI weighting matrices we have chosen $Q_{LQI} = \begin{bmatrix} 10^2 & 0 & 0 \\ 0 & 10^2 & 0 \\ 0 & 0 & 10^3 \end{bmatrix}$ and $R_{LQI} = 1$.

Finally, for H_∞ FI controller we have chosen the following templates which specifies the performance characteristics of the control problem: $\frac{1}{W_z}$ is a first order high pass filter having the bandwidth 0.4 rad/sec, the low frequencies attenuation -60 dB and high frequency amplification 4 dB; $\frac{1}{W_u}$ is a first order low pass filter having the bandwidth 14 rad/sec, the low frequencies amplification 10 dB and high frequency attenuation -40 dB.

B. Comparison between Control-based Observers designs

The results of the Control-based Observers for all three controllers are shown in Figure 4 for the state estimation and Figure 5 for the unknown input estimation. In Figure 3 a comparison between the noisy output of the system and the estimated output of the model is illustrated.

In order to be consistent in our comparison, the parameters of the controllers have been chosen such that all the observers have a similar speed of convergence. In addition, all simulations are made with the same initial conditions: $x(0) = [0.1, 0.2]^T$ and $\hat{x}(0) = [0.4, 0.5]^T$.

At a first glance it seems that using the robust H_∞ FI controller we get better results in terms of unknown input estimation as well as state estimation. The result isn't surprising since the robust controller is more complex than the other two optimal ones. To confirm the results the mean square error (MSE) between the real values and the estimated ones for both states and unknown input is given in Table I.

TABLE I: Mean Square Error real values and estimated ones

	MSE (x_1, $\hat{x}_1$)	MSE (x_2, $\hat{x}_2$)	MSE (v, $\hat{v}$)
CbO (LQR)	$1.2\ 10^{-4}$	$3.4\ 10^{-4}$	$63.6\ 10^{-3}$
CbO (LQI)	$1.4\ 10^{-4}$	$2.5\ 10^{-4}$	$5.5\ 10^{-3}$
CbO (H_∞ FI)	$2.5\ 10^{-4}$	$1.2\ 10^{-4}$	$1.9\ 10^{-3}$

Another discussion worth having is about the complexity of the observers in terms of computation load and dimensions of the model together with the designed controller. It can be noticed that the design of all three controllers reduces to solve an Algebraic Riccati Equation.

Thus the complexity of the observer is given by the dimension of the model used to solve the control problem. The dimension of the models for each control strategies used to design a CbO is summarized in Table II.

TABLE II: CbO dimensions for different control strategies

	CbO (LQR)	CbO (LQI)	CbO (H_∞ FI)
observer dim.	n	n + p	n + k

In particular, the Control-based Observer using the LQR controller has the smaller dimension model (the same state dimension as the observed system). Next, the case of the LQI controller the observer state dimension increases with the number of outputs (the integral action). Finally, in the case of Full Information H_∞ controller the complexity of the observer depends on the chosen performance specification, see equation (28). Thus the dimension of the observer increases with the size of the vector x_{ps}, which is k. This leads to the conclusion that the price for a 'good' estimation is paid in terms of increased complexity.

V. Conclusion

The paper has presented a comparison between different control strategies such as P, PI, LQR, LQI and H_∞ Full Information controller in order to illustrate the capabilities of a recently proposed technique to estimate the state and the unknown inputs of a dynamical system. It has been shown that using an H_∞ Full Information controller we obtain better results in terms of state and unknown input estimation than the other control strategies proposed, but with the cost of an increased complexity of the observer.

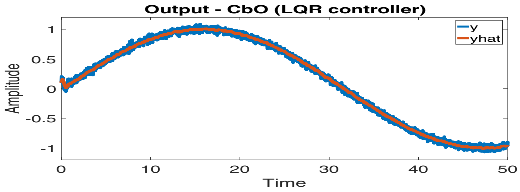
One of the main advantages of this method is that the unknown input estimation is directly obtained as a consequence of the control-based observer problem formulation. Since no assumption about the unknown inputs dynamic is made, the estimation results is better than in other classical methods.

It also shows that using this Control-based Observer paradigm the equations of more classical observers such as Luenberger Observer or PI Observer can be obtained.

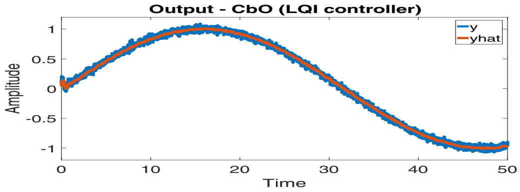
Extensions to more general forms of systems (nonlinear) are part of further work.

Acknowledgment

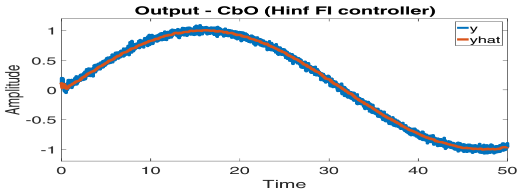
This work was partly supported by LabEx PERSYVAL-Lab (ANR-11- LABX-0025-01).



(a) Control-based Observer - LQR controller

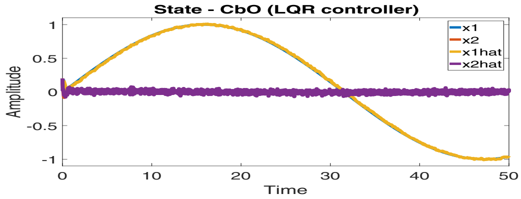


(b) Control-based Observer - LQI controller

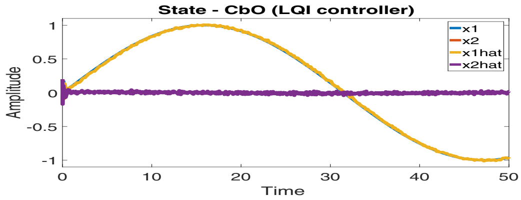


(c) Control-based Observer - H_∞ FI controller

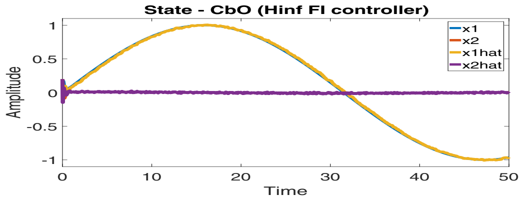
Fig. 3: System and model output



(a) Control-based Observer - QR controller

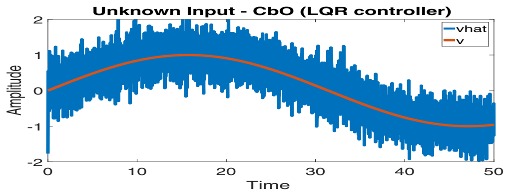


(b) Control-based Observer - LQI controller

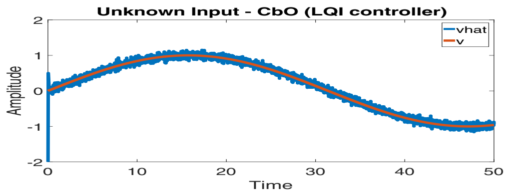


(c) Control-based Observer - H_∞ FI controller

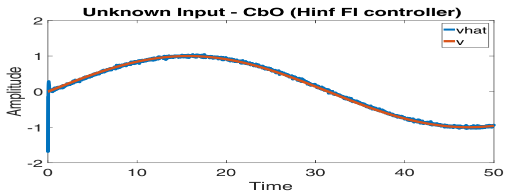
Fig. 4: State estimation



(a) Control-based Observer - LQR controller



(b) Control-based Observer - LQI controller



(c) Control-based Observer - H_∞ FI controller

Fig. 5: Unknown input estimation

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